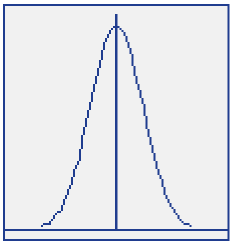
Citisoft
Global Investment Management Consultants

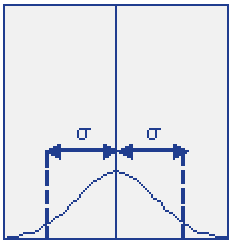
Volatility (σ) and Value at Risk (VaR)

What is Volatility (σ)?

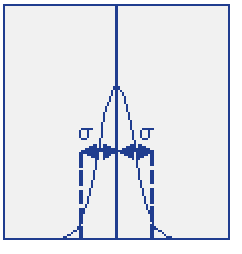
Volatility, or sigma, is a measure of how probable it is that a quantity value can differ wildly from its current value. Low volatility means that a quantity's value will be close to the current value most of the time; high volatility means that a quantity's value can reasonably be expected to vary by large amounts randomly over time. Volatility is defined as the standard deviation of the quantity over a defined time span. To understand this, consider the measurement of the daily return of a fund every day for one month. The daily returns will fall into intervals e.g. (1% → 1.1%), (1.1% → 1.2%), etc. At the end of the month there will be a certain number of daily returns in each interval. Dividing the numbers in each interval by the number of days in the month will give a measure of the normalised probability of the quantity being in that interval. The interval with the highest probability is called the mean value for the quantity. As a reasonable assumption, if the probabilities are plotted against, say, the mid points of the intervals, then the resulting curve would be a bell shaped normal distribution curve centred around the mean value as shown at the top on the right. The standard deviation is a statistical measure of the *spread* of the bell curve: a large standard deviation means that the curve is flat and spread out; a small standard deviation means the curve is tall and thin. If the curve is flat and spread out then there is a relatively high probability of the asset or fund return being in an interval some distance from the mean value and this means that the quantity is volatile; if the curve is tall and thin then most of the quantity values will be grouped closely around the mean value and the quantity has a low volatility.



Normal Distribution Curve



Large σ – High Volatility



Small σ – Low Volatility

Volatilities calculated on pure sample data that is assumed to be normally distributed are called **Normal Volatilities**. However, it is generally assumed in the market that asset prices are lognormally distributed, that is to say that if the frequencies of the natural logarithm of a price are plotted as described above, then the resulting probability curve has a normal distribution. This is an intrinsic assumption used in the Black-Scholes pricing model for options. For this reason, volatility can be expressed as the standard deviation of the logarithms of a quantity over a period, and such volatilities are called **Log Volatilities**. **Annual Volatility** is usually a normal volatility whereas **Annualised Volatility** (sic) is calculated and expressed as a log volatility.

How is Volatility Measured?

Volatility can be measured in two ways: **Historical Volatility** is calculated by analysing historical returns as described; **Implied Volatility** is determined by using Black-Scholes to calculate volatility from an option price published in the market. Implied volatilities are often referred to as a "market consensus" of volatility - an indication of risk that combines the insights of many market participants. Historic volatilities can suffer from bias depending on how the market was operating during the return sample period and in addition, the data on which they are based could be stale.

Another distinction when measuring volatility is how the mean value is treated in the calculation. In any two sample analyses, the mean value of the quantity will be different, and the standard deviation is always calculated relative to the mean. However, if the mean value is calculated, subtracted from each measurement, and then the analysis is repeated, the mean value will always be zero for any sample period. Volatilities calculated in this way have been found to be more stable than when calculated on a non-corrected basis.

Some practitioners argue that when calculating volatility, more weight should be given to recent returns. This leads to the concept of **Exponentially Weighted Volatilities**. In addition, doubt has been expressed as to whether the assumption that probability distributions fit the normal distribution curves is really valid. Other curves have been proposed which give more weight to the low probability – high

Volatility Equations

If y_i, i=1,….N are data samples over a period of time, say daily sample of annual returns, or daily prices, then the normal mean of the sample is calculated as:

$$\mu_{normal} = \frac{1}{N}\sum_{i=1}^{i=N} y_i$$

and the lognormal mean is calculated as:

$$\mu_{\text{lognormal}} = \frac{1}{N-1}\sum_{i=1}^{i=N-1} r_i$$

where

$$r_i = \frac{\log(y_{i+1})}{\log(y_i)}$$

The normal volatility is calculated as:

$$\sigma_{normal} = \sqrt{\frac{1}{N}\sum_{i=1}^{i=N}(y_i - \mu_{normal})^2}$$

and the lognormal volatility is calculated as:

$$\sigma_{\text{lognormal}} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{i=N}(r_i - \mu_{\text{lognormal}})^2}$$

Exponentially weighted normal volatility is calculated as:

$$\sigma_{normal} = \sqrt{\frac{1-\lambda}{1-\lambda^N}\sum_{i=1}^{i=N}\lambda^{i-1}(y_i - \mu_{normal})^2}$$

and exponentially weighted log normal volatility is calculated as:

$$\sigma_{\text{lognormal}} = \sqrt{\frac{1-\lambda}{1-\lambda^{N-1}}\sum_{i=1}^{i=N-1}\lambda^{i-1}(r_i - \mu_{\text{lognormal}})^2}$$

Covariance between any two data samples a_i, i = 1…N for which the mean is A and b_j, j= 1….N, for which the mean is B, is calculated as:

$$Cov(a,b) = \frac{1}{N}\sum_{i=1}^{i=N}(a_i - A)(b_i - B)$$

and this defines the Covariance Matrix.

If we denote the relative value of asset i in a fund by a vector ω, such that

$$\omega_i = \frac{V_i}{V_f}$$

then the return volatility of a fund of assets on which the covariance matrix has been constructed is given by:

$$\sigma_f = \sqrt{\omega^T Cov \omega}$$

The Variance / Co-Variance Method

If the relative current value of asset i in a fund is represented by a vector ω, such that

$$\omega_i = \frac{V_i}{V_f}$$

and μ_i are the expected (mean) values of asset i over the VaR time period, then the expected value of the fund is given by:

$$\mu_f = \sum_{i=1}^{N} \omega_i \mu_i$$

If σ_f is the volatility of the fund over the VaR time period is calculated as described on the previous page, then the VaR for the fund for a 95% confidence level is calculated from:

$$VaR = -V_f\left(\mu_f - 1.65\sigma_f\right)$$

For a 99% confidence level the formula becomes:

$$VaR = -V_f\left(\mu_f - 2.33\sigma_f\right)$$

If the returns are daily returns, then σ_f over period of D days can be calculated as:

$$\sigma_f(D) = \sigma_f\sqrt{D}$$

Which can be plugged into the above equations to calculate VaR for D days.

The Historical Method

If σ_f is the volatility of the fund over the VaR time period, T is calculated as described on the previous page, then the VaR for the fund for a 99% confidence level is calculated from:

$$VaR = 2.33M\sigma_f\sqrt{T}$$

where the factor 2.33 represents the number of σ_f required for a confidence level of 99%.

Citisoft Services

Citisoft is a specialist consulting firm focused on investment management. With a client base of more than 75% of the top 50 global investment managers, Citisoft is the foremost supplier of strategic planning and implementation services to the institutional, wealth management and alternative investment sectors.

Established in 1986, Citisoft offers complete end-to-end solutions. Although expert in all market systems, Citisoft is totally independent of all suppliers and service companies.

impact results. Such curves are said to have **fat tails** and their different shapes affect the volatility estimations.

Volatility is a critical concept in the calculation of Value at Risk (VaR) for a fund.

Value at Risk (VaR)

Value at Risk (VaR) is the maximum loss not exceeded with a given probability, defined as the confidence level, over a given period of time. The time period over which the loss probability has been calculated is usually related to the expected lifetime of a fund or the time it might reasonably be expected to take to liquidate the fund - time periods are typically quoted as 1 day, 10 days or 1 year. The confidence level is generally quoted as 100 minus the probability of a loss greater than the quoted VaR occurring within the time period. If a VaR calculation is quoted with a confidence level of 95%, then there is a 5% probability that a fund will lose more than the quoted VaR within the time horizon.

How is VaR Calculated?

The Variance / Co-Variance Method. This method requires historical variances and co-variances of the prices for the assets in a fund to be calculated. Providing the following assumptions hold:

- the change in the value of a fund is linearly dependent on (i.e., is a linear combination of) all the changes in the values of the assets, so that also the portfolio return is linearly dependent on all the asset returns;
- the asset returns are jointly normally distributed;

then a statistical formula can be used to calculate the VaR based on the volatility of the fund. The drawback of the method is the above set of assumptions.

The Historical Method

This method involves running a fund across a set of historical price changes to yield a distribution of changes in portfolio value, and computing a percentile (the VaR). The benefits of this method are its simplicity to implement, and the fact that it does not assume a normal distribution of asset returns. Drawbacks are the requirement for a large market database, and the computational intensity.

Monte Carlo Simulation

A Monte Carlo Simulation takes a set of inputs and based on the probability distributions i.e. the means and the variances, generates a large number of random values. These values are then passed through a model and the probability distributions of the outputs are constructed. This method can be used to calculate VaR. Monte Carlo is the most common method used to calculate VaR for funds containing derivatives, but its drawback is its complexity and the potentially large computational cost.

Current Challenges and Initiatives

Volatility and VaR are vital concepts in the risk management of portfolios and in financially uncertain times, good risk analysis and management becomes ever more important and of vital concern to clients.

One of the most important challenges facing fund managers is the incorporation of ever more complex instruments into the risk analyses of their portfolios. The explosion in the use of complex OTC derivatives and the increasing use of more obscure investments such as Corporate Loans, is seriously stretching the capabilities of established risk engines. Some risk system suppliers are grabbing the "bull by the horns" and incorporating these derivatives into their systems, but progress is slow and the playing field is by no means level.

Hedge funds are at the cutting edge when it comes to using complex derivative strategies and Citisoft has extensive experience in helping hedge funds to understand where their current system solution might be lacking and in helping them to assess and select alternative solutions. However, UCITS III means that even traditional long-only funds can now invest in complex derivatives and this has lead to the emergence of 130/30 funds based on derivative strategies.

Citisoft can help fund managers in a number of ways, including:

Education – Citisoft can help educate staff in OTC derivatives, other more complex instruments and in risk principles.

Risk Gap Analysis and System Selection – Citisoft can help fund managers assess their current risk regimes and to select alternative risk solutions.

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